OSNOVNA ŠKOLA

8. RAZRED

Nastavna tema:

SLIČNOST TROUGLOVA

REŠENJA – VEŽBANJE

Nivo: SVI

AUTOR: Bogdan Đurić

LEKCIJA:

TALESOVA TEOREMA

AUTOR: Bogdan Đurić

NIVO

2 – DOVOLJAN

AUTOR: Bogdan Đurić

REŠENJE

Kako su prave p i q paralelne, to za datu situaciju važi Talesova teorema, pa je:

$$AB:AD = BC:DE \Rightarrow x:6 = 4:8$$
$$\Rightarrow 8x = 24$$
$$\Rightarrow x = 3\ cm$$

AUTOR: Bogdan Đurić

REŠENJE

Kako su prave p i q paralelne, to za datu situaciju važi Talesova teorema, pa je:

$$AB:AD = AC:AE \Rightarrow x:2,5 = 8:4$$
$$\Rightarrow 4x = 20$$
$$\Rightarrow x = 5\ cm$$

AUTOR: Bogdan Đurić

REŠENJE

Kako su prave p i q paralelne, to za datu situaciju važi Talesova teorema, pa je:

$$AB:AD = BC:DE \Rightarrow x:4 = 4:4x$$
$$\Rightarrow 4x^2 = 16$$
$$\Rightarrow x^2 = 4$$
$$\Rightarrow x = 2\ cm$$

AUTOR: Bogdan Đurić

REŠENJE

Kako su prave AB i DE paralelne (zbog podudarnih uglova, to za datu situaciju važi Talesova teorema, pa je:

$$AD:BE = DC:EC \Longrightarrow x:12 = 6:9$$
$$\Longrightarrow 9x = 72$$
$$\Longrightarrow x = 8\ cm$$

AUTOR: Bogdan Đurić

REŠENJE

Kako su prave AB i DE paralelne, to za datu situaciju važi Talesova teorema, pa je:

$$AD:BE = DC:EC \Longrightarrow x:12 = 6:9$$
$$\Longrightarrow 9x = 72$$
$$\Longrightarrow x = 8\ cm$$

AUTOR: Bogdan Đurić

NIVO

3 – DOBAR

AUTOR: Bogdan Đurić

REŠENJE

Kako su prave c, d i e paralelne, to za datu situaciju važi Talesova teorema, pa je:

$$\left.\begin{matrix} AB:AE = BC:EF \\ AB:AE = CD:FG \end{matrix}\right\} \Rightarrow \begin{cases} 10:5 = 4:x \\ 10:5 = 8:y \end{cases}$$

$$\Rightarrow \begin{cases} x = 2 \\ y = 4 \end{cases}$$

$$\Rightarrow x \cdot y = 8$$

AUTOR: Bogdan Đurić

REŠENJE

Kako su prave c, d i e paralelne, to za datu situaciju važi Talesova teorema, pa je:

$$\left.\begin{matrix} SA_1 : SB_1 = A_2A_3 : B_2B_3 \\ SA_2 : SB_2 = A_2A_3 : B_2B_3 \end{matrix}\right\} \Rightarrow \begin{cases} 6 : x = 4 : 2 \\ 3 : y = 4 : 2 \end{cases}$$

$$\Rightarrow \begin{cases} x = 3\ cm \\ y = 1,5\ cm \end{cases}$$

$$\Rightarrow {}^{x}/_{y} = 2$$

AUTOR: Bogdan Đurić

REŠENJE

Kako su prave d i e paralelne, to za datu situaciju važi Talesova teorema, pa je:

$$\left.\begin{matrix} 2x:12 = 12:18 \\ 4x:12 = y:18 \end{matrix}\right\} \Rightarrow \begin{cases} 2x = 8 \\ y = 6x \end{cases}$$

$$\Rightarrow \begin{cases} x = 4\ cm \\ y = 24\ cm \end{cases}$$

$$\Rightarrow x + y = 28\ cm$$

AUTOR: Bogdan Đurić

REŠENJE

Kako su prave d i e paralelne, to za datu situaciju važi Talesova teorema, pa je:

$$\left.\begin{matrix} x:12 = 24:16 \\ y:12 = 24:18 \end{matrix}\right\} \Rightarrow \begin{cases} 16x = 12 \cdot 24 \\ 18y = 12 \cdot 24 \end{cases}$$

$$\Rightarrow \begin{cases} x = 18\ cm \\ y = 16\ cm \end{cases}$$

$$\Rightarrow x + y = 34\ cm$$

AUTOR: Bogdan Đurić

REŠENJE

Kako su prave d i e paralelne, to za datu situaciju važi Talesova teorema, pa je:

$$\left.\begin{matrix} x:12=(x+6):16 \\ y:16=18:(x+6) \end{matrix}\right\} \Rightarrow \begin{cases} 16x=12x+12\cdot 6 \\ y\cdot(x+6)=16\cdot 18 \end{cases}$$

$$\Rightarrow \begin{cases} x=18\ cm \\ y=12\ cm \end{cases}$$

$$\Rightarrow x+y=30\ cm$$

AUTOR: Bogdan Đurić

NIVO

4 – VRLO DOBAR

AUTOR: Bogdan Đurić

REŠENJE

Kako je $p \parallel BC$, to za datu situaciju važi Talesova teorema, pa je za date podatke:

$$\left.\begin{matrix} DC:EB = AC:AB \\ EB = AD \end{matrix}\right\} \Longrightarrow DC:AD = 10:15 \Bigg\} \Longrightarrow$$
$$AD + DC = 10\ cm$$

$$\Longrightarrow \begin{cases} DC = \dfrac{10}{15} AD \\ AD + \dfrac{10}{15} AD = 10\ cm \end{cases} \Longrightarrow AD = 6\ cm$$

AUTOR: Bogdan Đurić

REŠENJE

Kako je $p \parallel BC$, to za datu situaciju važi Talesova teorema, pa je za date podatke:

$$\left.\begin{matrix} AC:AD = AB:AE \\ AB:AE = 5:3 \end{matrix}\right\} \Longrightarrow AC:AD = 5:3 \Bigg\} \Longrightarrow$$
$$AD + DC = AC$$

$$\Longrightarrow \begin{cases} AC = \dfrac{5}{3}AD \\ AD + 4\ cm = \dfrac{5}{3}AD \end{cases} \Longrightarrow AD = 6\ cm$$

AUTOR: Bogdan Đurić

REŠENJE

Kako je $p \parallel BC$, to za datu situaciju važi Talesova teorema, pa je za date podatke:

$$\left.\begin{matrix} AC:AD = AB:AE \\ AB:AE = 5:3 \end{matrix}\right\} \Longrightarrow AC:AD = 5:3 \Bigg\} \Longrightarrow$$
$$AD + DC = AC$$

$$\Longrightarrow \begin{cases} AC = \dfrac{5}{3}AD \\ AD + AD - 2\ cm = \dfrac{5}{3}AD \end{cases} \Longrightarrow AD = 6\ cm$$

AUTOR: Bogdan Đurić

REŠENJE

Kako su prave AM, BN, CP i DQ međusobno paralelne, to za datu situaciju važi Talesova teorema, pa je za date podatke:

$$\left.\begin{array}{l} OM:OA = OP:OC \\ OP = OD = 18 \end{array}\right\} \Rightarrow OM:5 = 18:12$$

$$\Rightarrow OM = 7,5$$

$$\Rightarrow 2 \cdot OM = 15$$

AUTOR: Bogdan Đurić

REŠENJE

Kako su prave AM, BN, CP i DQ međusobno paralelne, to za datu situaciju važi Talesova teorema, pa je za date podatke:

$$\left.\begin{array}{c} ON:OB = PQ:CD \\ ON = PQ + 3 \\ OM:OA = PQ:CD \end{array}\right\} \Rightarrow \begin{cases} (PQ+3):8 = PQ:6 \\ OM:5 = PQ:6 \end{cases}$$

$$\Rightarrow \begin{cases} PQ = 9 \\ OM = 7,5 \end{cases}$$

$$\Rightarrow 2 \cdot OM = 15$$

AUTOR: Bogdan Đurić

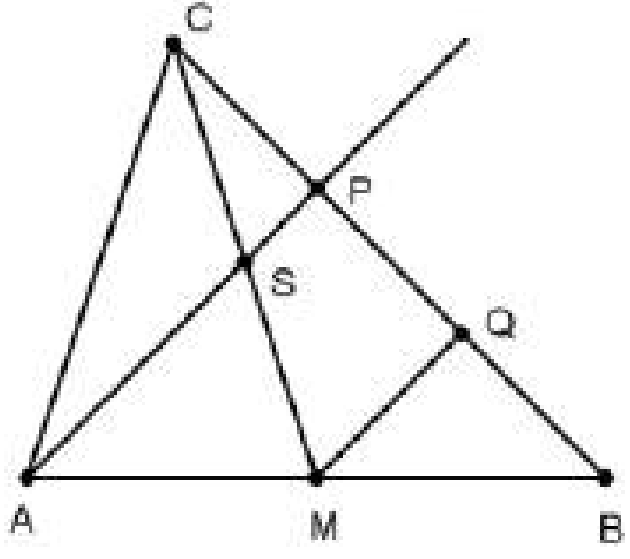
NIVO

5 – ODLIČAN

AUTOR: Bogdan Đurić

REŠENJE

Konstruišimo duž MQ tako da su prave AP i MQ međusobno paralelne. Za datu situaciju važi Talesova teorema, pa je za date podatke:



$$MQ \parallel AP (\parallel SP) \Longrightarrow$$

$$\Longrightarrow \begin{cases} CP : PQ = CS : SM = 1 : 1 \\ BQ : PQ = BM : AM = 1 : 1 \end{cases}$$

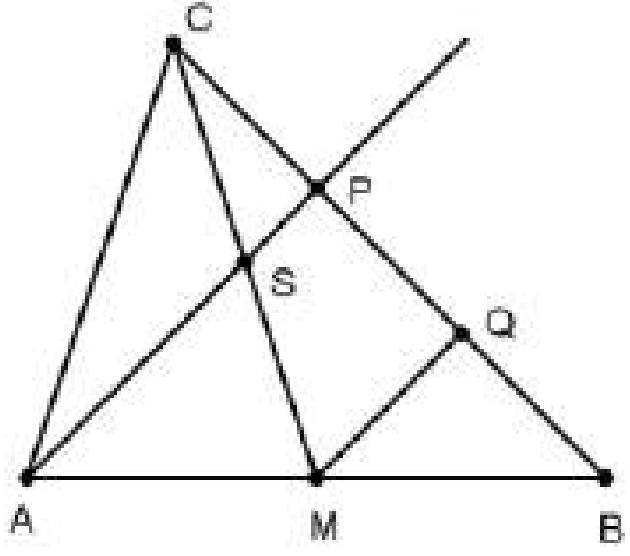
$$\Longrightarrow CP = PQ = QB = 2\ cm$$

$$\Longrightarrow PB = 4\ cm$$

AUTOR: Bogdan Đurić

REŠENJE

Konstruišimo duž MQ tako da su prave AP i MQ međusobno paralelne. Za datu situaciju važi Talesova teorema, pa je za date podatke:



$$MQ \parallel AP (\parallel SP) \Longrightarrow$$

$$\Longrightarrow \begin{cases} CP : PQ = CS : SM = 1 : 1 \\ BQ : PQ = BM : AM = 1 : 1 \end{cases}$$

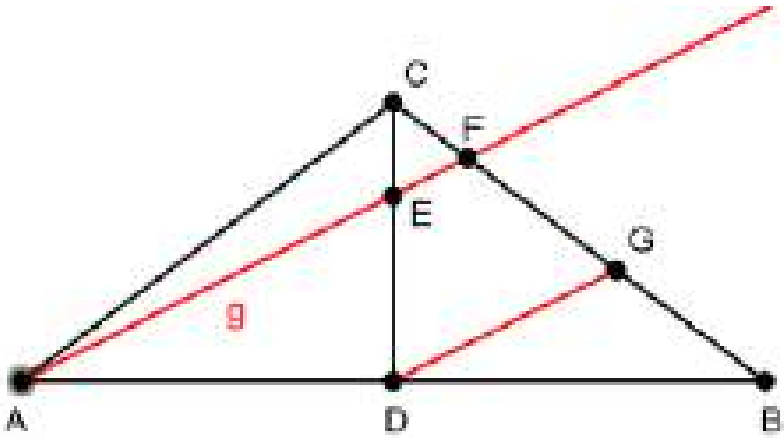
$$\Longrightarrow CP = PQ = QB = \frac{1}{3} BC$$

$$\Longrightarrow PB = 8\ cm$$

AUTOR: Bogdan Đurić

REŠENJE

Konstruišimo duž DG tako da su prave AF i DG međusobno paralelne. Za datu situaciju važi Talesova teorema, pa je za date podatke:



$$DG \parallel AF (\parallel EF) \Longrightarrow$$

$$\Longrightarrow \begin{cases} CF : FG = CE : ED = 1 : 2 \\ BG : FG = BD : AD = 1 : 1 \end{cases}$$

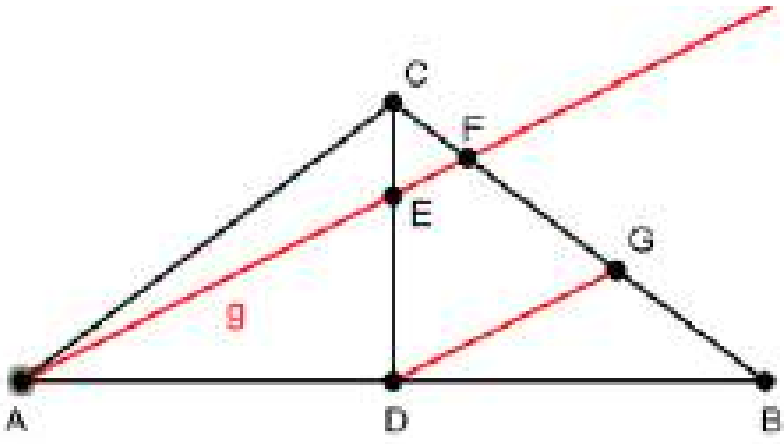
$$\Longrightarrow 2CF = FG = GB = 4\ cm$$

$$\Longrightarrow FB = 8\ cm$$

AUTOR: Bogdan Đurić

REŠENJE

Konstruišimo duž DG tako da su prave AF i DG međusobno paralelne. Za datu situaciju važi Talesova teorema, pa je za date podatke:



$$DG \parallel AF (\parallel EF) \Longrightarrow$$

$$\Longrightarrow \begin{cases} CF : FG = CE : ED = 1 : 2 \\ BG : FG = BD : AD = 1 : 1 \end{cases}$$

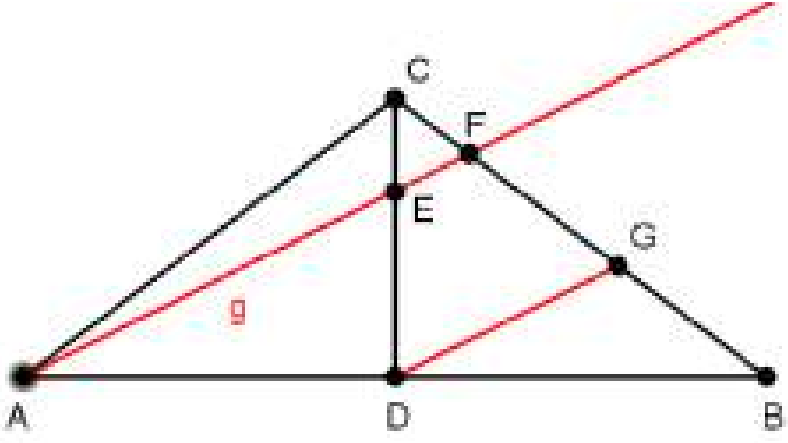
$$\Longrightarrow CF : FB = 1 : 4$$

$$\Longrightarrow FB = \frac{4}{5} BC = 16\ cm$$

AUTOR: Bogdan Đurić

REŠENJE

Konstruišimo duž DG tako da je $AF \parallel DG$. Za datu situaciju važi Talesova teorema, pa je za date podatke:



$$DG \parallel AF (\parallel EF) \Longrightarrow$$

$$\Longrightarrow \begin{cases} BD : AD = BG : FG = 1 : 1 \\ CF : FG = CF : \frac{1}{2} FB = 1 : 2 \\ CE : ED = CF : FG \end{cases}$$

$$\Longrightarrow CE : ED = 1 : 2$$

$$\Longrightarrow DE = \frac{2}{3} CD = 8\ cm$$

AUTOR: Bogdan Đurić

LEKCIJA:

SLIČNOST TROUGLOVA

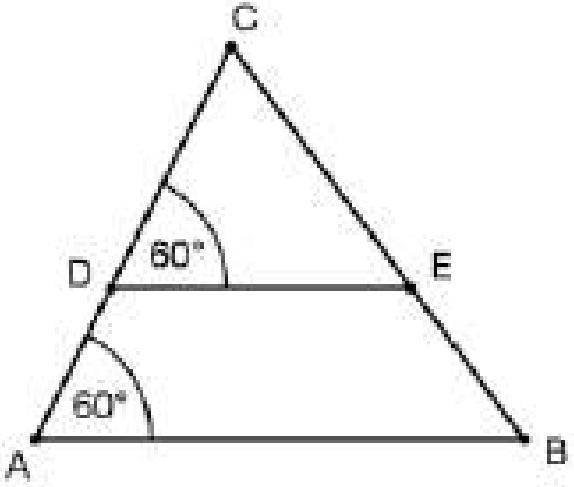
AUTOR: Bogdan Đurić

NIVO

2 – DOVOLJAN

AUTOR: Bogdan Đurić

REŠENJE



Dati trouglovi jesu slični jer je:

$$\left.\begin{array}{l} \sphericalangle BAC \cong \sphericalangle EDC \\ \sphericalangle ACB \cong \sphericalangle DCE \end{array}\right\} \overset{2.st.\ (UU)}{\Longrightarrow} \Delta ABC \sim \Delta DEC$$

AUTOR: Bogdan Đurić

REŠENJE

Prema datim podacima imamo:

$$\left.\begin{array}{l} \gamma_1 = \gamma = 87^\circ \\ \alpha_1 + \beta_1 + \gamma_1 = 180^\circ \end{array}\right\} \Rightarrow$$

$$\Rightarrow \alpha_1 + \beta_1 = 180^\circ - 87^\circ$$

$$\Rightarrow \alpha_1 + \beta_1 = 93^\circ$$

AUTOR: Bogdan Đurić

REŠENJE

Prema datim podacima imamo:

$$\left.\begin{array}{l} \gamma_1 = \gamma = 90^\circ \\ \alpha_1 + \beta_1 + \gamma_1 = 180^\circ \end{array}\right\} \Longrightarrow$$

$$\Longrightarrow \alpha_1 + \beta_1 = 180^\circ - 90^\circ$$

$$\Longrightarrow \alpha_1 + \beta_1 = 90^\circ$$

AUTOR: Bogdan Đurić

REŠENJE

Prema datim podacima imamo:

$$\left.\begin{array}{l} \mathcal{O} = a + b + c = 15\ cm \\ b_1 = 7b \Longrightarrow b_1 : b = 7:1 \\ \qquad \mathcal{O}_1 : \mathcal{O} = b_1 : b \end{array}\right\} \Longrightarrow$$

$$\Longrightarrow \mathcal{O}_1 : 15 = 7:1$$

$$\Longrightarrow \mathcal{O}_1 = 105\ cm$$

AUTOR: Bogdan Đurić

REŠENJE

Prema datim podacima imamo:

$$\left.\begin{array}{r} P = \dfrac{a \cdot b}{2} = 7,5\ cm^2 \\ b_1 = 2b \Longrightarrow b_1 : b = 2:1 \\ P_1 : P = b_1^2 : b^2 \end{array}\right\} \Longrightarrow P_1 : 7,5 = 4:1$$

$$\Longrightarrow P_1 = 30\ cm^2$$

AUTOR: Bogdan Đurić

NIVO

3 – DOBAR

AUTOR: Bogdan Đurić

REŠENJE

Prema datim podacima imamo:

$$\left.\begin{array}{l} \sphericalangle BAC \cong \sphericalangle DAE \\ \sphericalangle ACB \cong \sphericalangle AED \end{array}\right\} \xRightarrow{2.st.(UU)} \Delta ABC \sim \Delta ADE$$

$$\Rightarrow AE : AC = DE : BC$$

$$\Rightarrow AE : 20 = 6 : 10$$

$$\Rightarrow AE = 12\ cm$$

AUTOR: Bogdan Đurić

REŠENJE

Prema datim podacima imamo:

$$\left.\begin{array}{l} \sphericalangle BAC \cong \sphericalangle DAE \\ \sphericalangle ACB \cong \sphericalangle AED \end{array}\right\} \xrightarrow{2.st.(UU)} \Delta ABC \sim \Delta ADE$$

$$\Longrightarrow AE : AC = DE : BC$$

$$\Longrightarrow AE : (AE + 8) = 6 : 10$$

$$\Longrightarrow AE = 12\ cm$$

AUTOR: Bogdan Đurić

REŠENJE

Prema datim podacima imamo:

$$\left.\begin{matrix} c^2 = a^2 + b^2 \\ f : c = d : a \end{matrix}\right\} \Longrightarrow \begin{cases} c = 10\ cm \\ f : 10 = 3 : 6 \end{cases}$$

$$\Longrightarrow f = 5\ cm$$

AUTOR: Bogdan Đurić

REŠENJE

Prema datim podacima imamo:

$$k = 2 \Longrightarrow \begin{cases} a_1 = 2a \\ b_1 = 2b \end{cases}$$

$$\Longrightarrow \begin{cases} a_1 = 16\ cm \\ b_1 = 20\ cm \end{cases}$$

$$\Longrightarrow a_1 + b_1 = 36\ cm$$

AUTOR: Bogdan Đurić

REŠENJE

Prema datim podacima imamo:

$$\left.\begin{array}{r} a = 45\ cm \wedge a_1 = 27\ cm \\ (b_1 + c_1 + d_1) : (b + c + d) = a_1 : a \end{array}\right\} \Longrightarrow$$

$$\Longrightarrow (b_1 + c_1 + d_1) : 65 = 27 : 45$$
$$\Longrightarrow b_1 + c_1 + d_1 = 39\ cm$$

AUTOR: Bogdan Đurić

NIVO

4 – VRLO DOBAR

AUTOR: Bogdan Đurić

REŠENJE

Prema datim podacima imamo:

$$\left.\begin{array}{l} \sphericalangle AEB \cong \sphericalangle CED \\ \sphericalangle BAE \cong \sphericalangle DCE \end{array}\right\} \xRightarrow{2.st.(UU)} \Delta ABE \sim \Delta CDE$$

$$\Rightarrow AB : CD = AE : CE$$

$$\Rightarrow AB : 15 = 9 : 5$$

$$\Rightarrow AB = 27\ cm$$

AUTOR: Bogdan Đurić

REŠENJE

Prema datim podacima imamo:

$$\left.\begin{aligned} h_{a_1}^2 &= b_1^2 - \left(\frac{a_1^2}{2}\right)^2 \\ a_2 : a_1 &= h_{a_2} : h_{a_1} \end{aligned}\right\} \Rightarrow \begin{cases} h_{a_1} = 6\ cm \\ a_2 : 16 = 9 : 6 \end{cases}$$

$$\Rightarrow a_2 = 24\ cm$$

$$\Rightarrow P_2 = \frac{a_2 \cdot h_{a_2}}{2} = 108\ cm^2$$

AUTOR: Bogdan Đurić

REŠENJE

Prema datim podacima imamo:

$$\left.\begin{array}{l} \sphericalangle BAC \cong \sphericalangle EDC \\ \sphericalangle ACB \cong \sphericalangle DCE \end{array}\right\} \overset{2.st.(UU)}{\Longrightarrow} \Delta ABC \sim \Delta DEC$$

$$\Longrightarrow AC : DC = BC : EC$$

$$\Longrightarrow (x+6) : 8 = 12 : 6$$

$$\Longrightarrow x = 10\ cm$$

AUTOR: Bogdan Đurić

REŠENJE

Prema datim podacima imamo:

$$\left.\begin{array}{l} \sphericalangle ACD \sim \sphericalangle CBD \\ (2.st. - UU) \end{array} \Longrightarrow CD = \sqrt{AD \cdot BD} = 6\ cm \right\} \Longrightarrow$$

$$P = \frac{AB \cdot CD}{2}$$

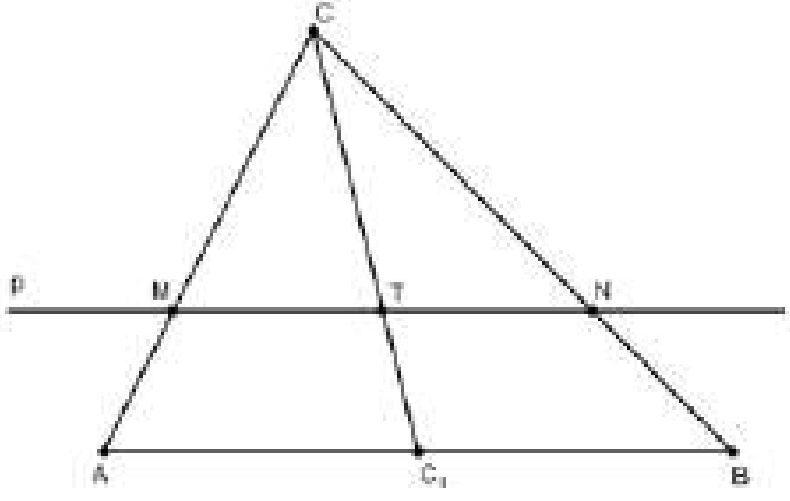
$$\Longrightarrow P = 39\ cm^2$$

AUTOR: Bogdan Đurić

REŠENJE

$$\left.\begin{array}{l} \Delta ABC \sim \Delta MNC \\ CT : CC_1 = 2:3 \\ \mathcal{O}_1 : \mathcal{O} = CT : CC_1 \end{array}\right\} \Rightarrow \mathcal{O}_1 : \mathcal{O} = 2:3 \Bigg\} \Rightarrow$$

$$\mathcal{O} = a + b + c = 81\ cm$$

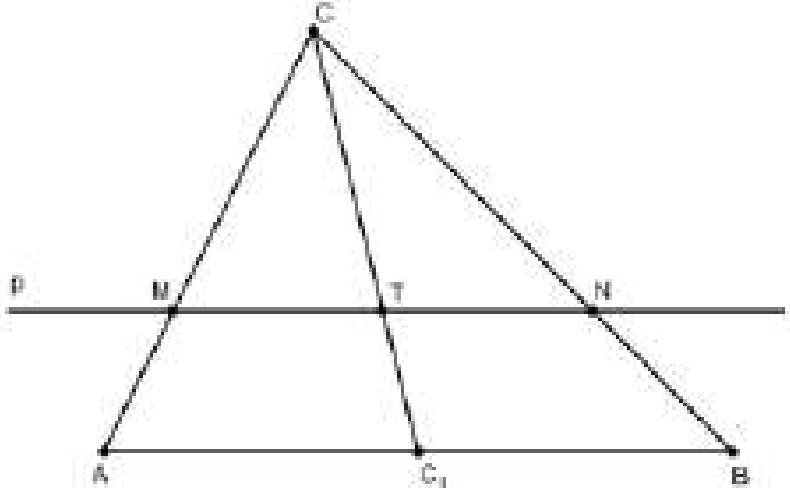


$$\Rightarrow \mathcal{O}_1 = 54\ cm$$

AUTOR: Bogdan Đurić

REŠENJE

$$\left.\begin{array}{r} \Delta ABC \sim \Delta MNC \\ CT : CC_1 = 2:3 \\ P_{\Delta MNC} : P_{\Delta ABC} = CT^2 : CC_1^2 = 4:9 \\ P_T = P_{\Delta ABC} - P_{\Delta MNC} \end{array}\right\} \Rightarrow P_T : P_{\Delta ABC} = 5:9$$



$$\Rightarrow P_T : 36 = 5:9$$
$$\Rightarrow P_T = 20\ cm^2$$

AUTOR: Bogdan Đurić

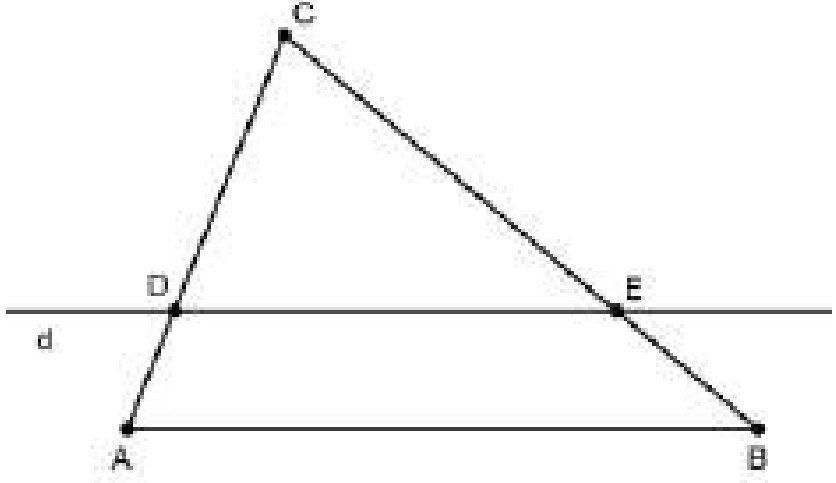
NIVO

5 – ODLIČAN

AUTOR: Bogdan Đurić

REŠENJE

$$\left.\begin{aligned} P_{\Delta DEC} : P_{\Delta ABC} &= 1:2 \\ P_{\Delta DEC} : P_{\Delta ABC} &= DE^2 : AB^2 \end{aligned}\right\} \Rightarrow$$



$$\Rightarrow DE^2 : \left(7\sqrt{2}\right)^2 = 1:2$$

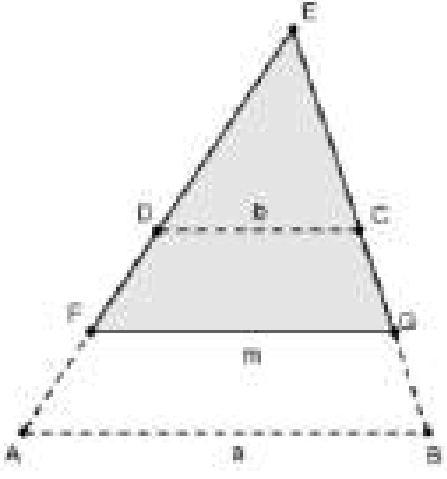
$$\Rightarrow DE = 7\ cm$$

AUTOR: Bogdan Đurić

REŠENJE

$$m = \frac{a+b}{2} = 3\ cm$$

$$\left.\begin{array}{l} \Delta ABE \sim \Delta FGE \Longrightarrow P_1 : P_{\Delta ABE} = m^2 : a^2 = 9:16 \\ P_2 = P_{\Delta ABE} - P_1 \end{array}\right\} \Longrightarrow$$



$$\Longrightarrow P_1 : P_2 = 9:7$$
$$\Longrightarrow P_1 : 7 = 9:7$$
$$\Longrightarrow P_1 = 9\ cm^2$$

AUTOR: Bogdan Đurić

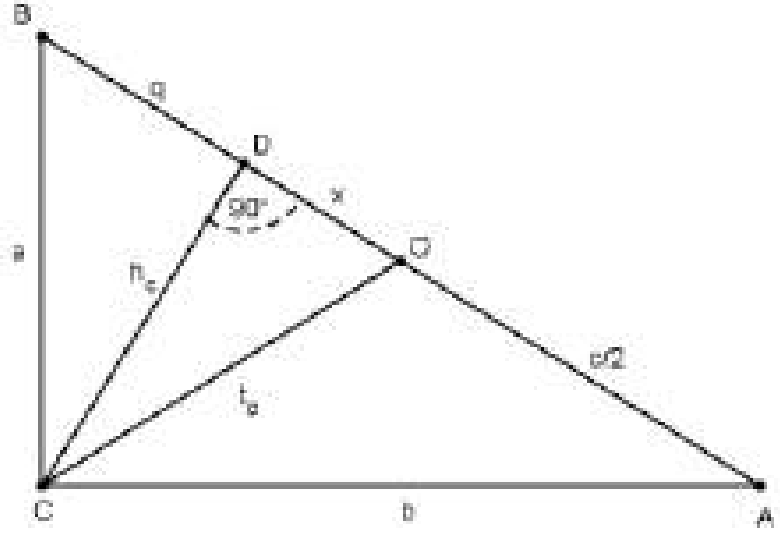
REŠENJE

$$\left.\begin{array}{l} a : b = h_b : h_a \Longrightarrow b = \dfrac{4}{3}a \Longrightarrow b_1 = \dfrac{4}{3}a_1 \\ \mathcal{O}_1 = a_1 + 2b_1 = 22\ cm \end{array}\right\} \Longrightarrow$$

$$\Longrightarrow a_1 = 6\ cm$$

AUTOR: Bogdan Đurić

REŠENJE



$$x^2 = t_c^2 - h_c^2 \Rightarrow x = 3,5\ cm$$

$$q = \frac{c}{2} - x \Rightarrow q = 9\ cm$$

$$a^2 = h_c^2 + q^2 \Rightarrow a = 15\ cm$$

$$c = 2t_c \Rightarrow c = 25\ cm$$

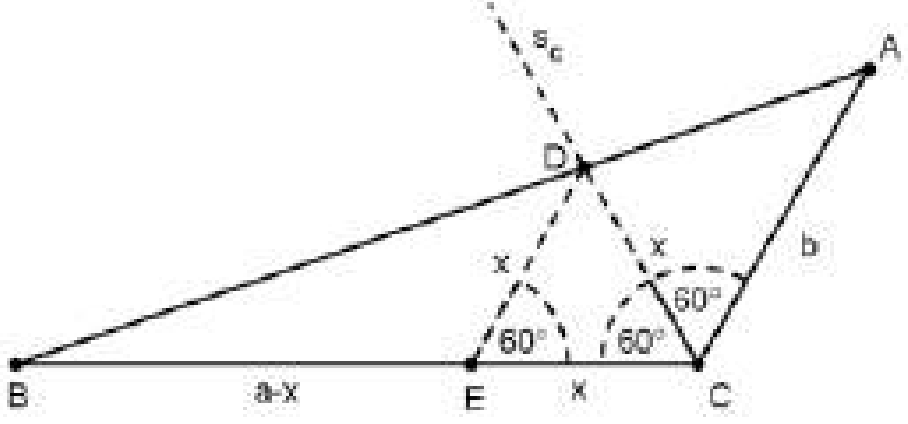
$$b^2 = c^2 - a^2 \Rightarrow b = 20\ cm$$

$$\mathcal{O} = a + b + c \Rightarrow \mathcal{O} = 60\ cm$$

AUTOR: Bogdan Đurić

REŠENJE

$$DE \parallel AC \Longrightarrow \begin{cases} \Delta CDE \text{ jednakostraničan} \\ \Delta DBE \sim \Delta ABC \end{cases}$$



$$\Longrightarrow (a - x) : a = x : b$$
$$\Longrightarrow (12 - x) : 12 = x : 6$$
$$\Longrightarrow CD = x = 4\ cm$$

AUTOR: Bogdan Đurić

REŠENJE

$$h_1 = h_2 = h_3 = h_4 = \frac{h}{4} \Longrightarrow$$

$$\Longrightarrow \begin{cases} P_1 : P = h_1^2 : h^2 = 1 : 16 \\ (P_1 + P_2) : P = (h_1 + h_2)^2 : h^2 = 4 : 16 \\ (P_1 + P_2 + P_3) : P = (h_1 + h_2 + h_3)^2 : h^2 = 9 : 16 \end{cases}$$

$$\Longrightarrow \left\{ P_1 = \frac{1}{16} P \wedge P_2 = \frac{3}{16} P \wedge P_3 = \frac{5}{16} P \wedge P_4 = \frac{7}{16} P \right.$$

$$\Longrightarrow P_4 - P_3 + P_2 - P_1 = \frac{4}{16} P = 4\ cm^2$$

AUTOR: Bogdan Đurić

LEKCIJA:

PRIMENA SLIČNOSTI NA PRAVOUGLI TROUGAO

AUTOR: Bogdan Đurić

NIVO

2 – DOVOLJAN

AUTOR: Bogdan Đurić

REŠENJE

Topola i njena senka, kao i štap i njegova senka, formiraju pravougle trouglove. Kako se može smatrati da su zraci Sunčeve svetlosti paralelni, tada su ta dva trougla slična (2. stav – UU), pa je visina topole:

$$h : 20 = 2 : 1,6 \Longrightarrow h = 25\ m$$

AUTOR: Bogdan Đurić

REŠENJE

Betonski stub i njegova senka, kao i štap i njegova senka, formiraju pravougle trouglove. Kako se može smatrati da su zraci Sunčeve svetlosti paralelni, tada su ta dva trougla slična (2. stav – UU), pa je dužina senke štapa:

$$\ell : 120 = 12 : 8 \Rightarrow \ell = 180\ cm$$

AUTOR: Bogdan Đurić

REŠENJE

Trougao na karti i trougao u prirodi su slični (zbog razmere), pa je:

$$\left.\begin{array}{c} \mathcal{O}_1 = 14 + 10 + 12 = 36\ cm \\ \mathcal{O}_1 : \mathcal{O} = 1 : 500\ 000 \end{array}\right\} \Longrightarrow$$

$$\Longrightarrow \mathcal{O} = 180\ km$$

AUTOR: Bogdan Đurić

REŠENJE

Trougao na karti i trougao u prirodi su slični (zbog razmere), pa je:

$$\left.\begin{array}{c} P = 8\ cm^2 \\ P : P_1 = 1^2 : 500\ 000^2 \end{array}\right\} \Longrightarrow$$

$$\Longrightarrow P_1 = 200\ km^2$$

AUTOR: Bogdan Đurić

REŠENJE

Stub dalekovoda i njegova senka, kao i Aleksandar i njegova senka, formiraju pravougle trouglove. Kako se može smatrati da su zraci Sunčeve svetlosti paralelni, tada su ta dva trougla slična (2. stav – UU), pa je visina stuba:

$$\left.\begin{array}{r} \ell = 25\,m \\ h:\ell = h_A:\ell_A = 1:1 \end{array}\right\} \Rightarrow h = 25\,m$$

AUTOR: Bogdan Đurić

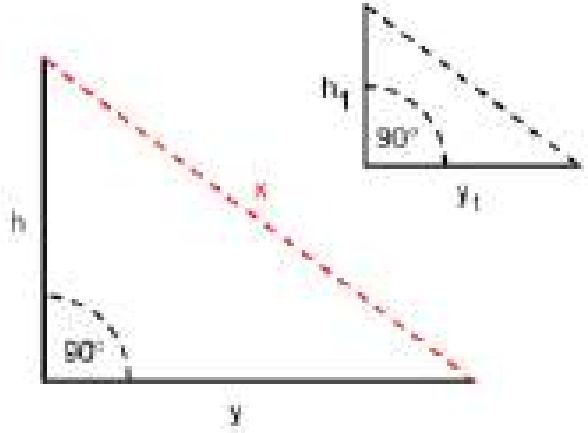
NIVO

3 – DOBAR

AUTOR: Bogdan Đurić

REŠENJE

Topola i njena senka, kao i štap i njegova senka, formiraju pravougle trouglove. Kako se može smatrati da su zraci Sunčeve svetlosti paralelni, tada su ta dva trougla slična (2. stav – UU), pa je:



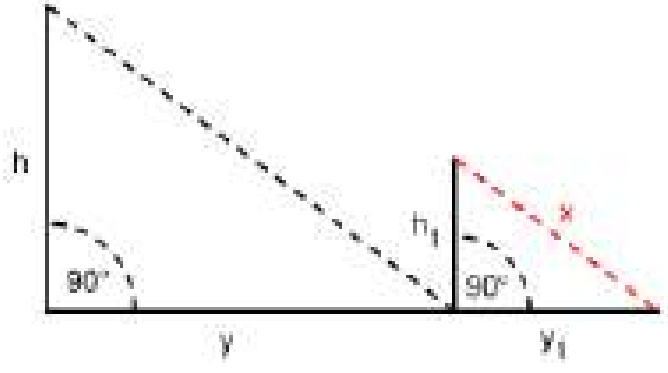
$$h : h_1 = y : y_1 \Rightarrow h : 1,5 = 8 : 2$$
$$\Rightarrow h = 6\ m$$

$$x^2 = h^2 + y^2 \Rightarrow x^2 = 6^2 + 8^2$$
$$\Rightarrow x = 10\ m$$

AUTOR: Bogdan Đurić

REŠENJE

Betonski stub i njegova senka, kao i štap i njegova senka, formiraju pravougle trouglove. Kako se može smatrati da su zraci Sunčeve svetlosti paralelni, tada su ta dva trougla slična (2. stav – UU), pa je:



$$h_1 : h = y_1 : y \Rightarrow 150 : 6 = y_1 : 8$$
$$\Rightarrow y_1 = 200\ cm$$

$$x^2 = h_1^2 + y_1^2 \Rightarrow x^2 = 150^2 + 200^2$$
$$\Rightarrow x = 250\ cm$$

AUTOR: Bogdan Đurić

REŠENJE

Trougao na karti i trougao u prirodi su slični (zbog razmere), pa je:

$$\left.\begin{array}{c} b^2 = c^2 - a^2 \Longrightarrow b = 8\ cm \\ \mathcal{O}_1 = a + b + c \Longrightarrow \mathcal{O}_1 = 24\ cm \\ \mathcal{O}_1 : \mathcal{O} = 1 : 800\,000 \end{array}\right\} \Longrightarrow$$

$$\Longrightarrow \mathcal{O} = 192\ km$$

AUTOR: Bogdan Đurić

REŠENJE

Trougao na karti i trougao u prirodi su slični (zbog razmere), pa je:

$$\left.\begin{array}{l} b^2 = c^2 - a^2 \Longrightarrow b = 8\ cm \\ P_1 = \dfrac{a \cdot b}{2} \Longrightarrow P_1 = 24\ cm^2 \\ P_1 : P = 1^2 : 650\,000^2 \end{array}\right\} \Longrightarrow$$

$$\Longrightarrow P = 1014\ km^2$$

AUTOR: Bogdan Đurić

REŠENJE

Stub dalekovoda i njegova senka, kao i Aleksandar i njegova senka, formiraju pravougle trouglove. Kako se može smatrati da su zraci Sunčeve svetlosti paralelni, tada su ta dva trougla slična (2. stav – UU), pa je visina stuba:

$$\left.\begin{array}{r} \ell = 25\ m \\ h : \ell = h_A : \ell_A = 1 : 1\frac{1}{4} \end{array}\right\} \Longrightarrow h = 20\ m$$

AUTOR: Bogdan Đurić

NIVO

4 – VRLO DOBAR

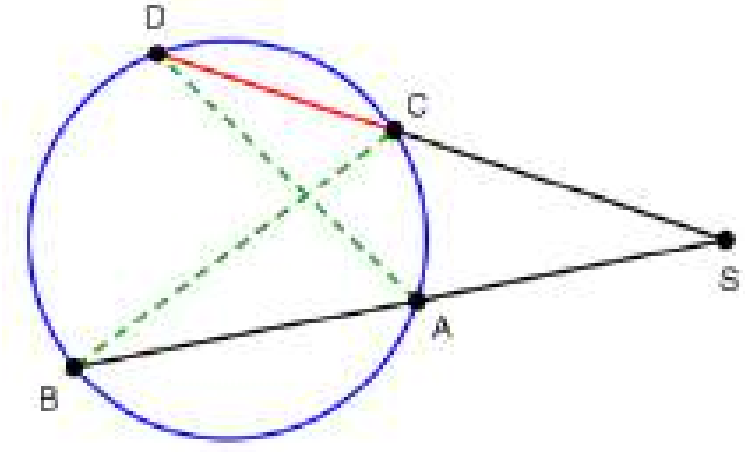
AUTOR: Bogdan Đurić

REŠENJE

Na osnovu crteža i datih podataka imamo da je:

$$\left.\begin{array}{l} \sphericalangle DSA \cong \sphericalangle BSC \\ (zajednički) \\ \sphericalangle ADS \cong \sphericalangle CBS \\ (perif.nad\ \widehat{AC}) \end{array}\right\} \xrightarrow{2.st.(UU)} \Delta SDA \cong \Delta SBC$$

$$\Rightarrow SD:SB = SA:SC$$
$$\Rightarrow (18 + CD):36 = 16:18$$
$$\Rightarrow CD = 14\ cm$$



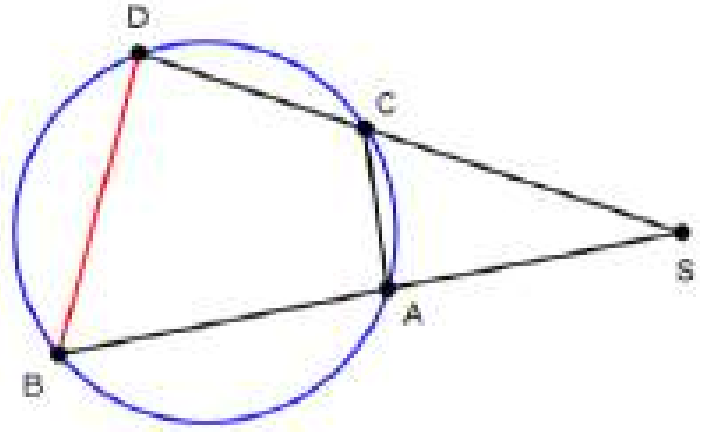
AUTOR: Bogdan Đurić

REŠENJE

Uočimo da su periferijski uglovi kod temena D i A nad suprotnim lukovima ($\widehat{BC}$ i $\widehat{CB}$), te su suplementni, pa je:

$$\left.\begin{array}{l} \sphericalangle BDC + \sphericalangle CAB = 180° \\ \sphericalangle CAS + \sphericalangle CAB = 180° \end{array}\right\} \Rightarrow \sphericalangle BDS \cong \sphericalangle CAS \left.\begin{array}{l} \\ \\ \\ \end{array}\right\} \overset{2.st.(UU)}{\Longrightarrow}$$

$$\sphericalangle DSB \cong \sphericalangle ASC \ (zajed.)$$



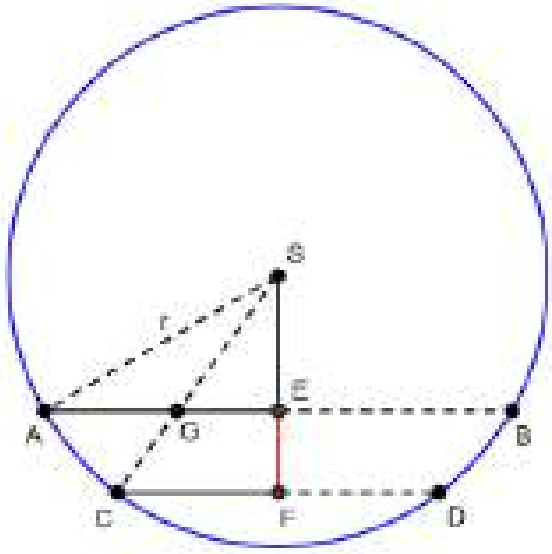
$$\Rightarrow \Delta SDB \cong \Delta SAC$$
$$\Rightarrow BD : CA = SB : SC$$
$$\Rightarrow BD : 6 = 15 : 10$$
$$\Rightarrow BD = 9\ cm$$

AUTOR: Bogdan Đurić

REŠENJE

Na osnovu crteža i datih podataka, imamo da je:

$$\left.\begin{array}{l} AE = 4\ cm \wedge r^2 = AE^2 + SE^2 \Rightarrow r = 5\ cm \\ CF = 3\ cm \wedge SF^2 = r^2 - CF^2 \end{array}\right\} \Rightarrow SF = 4\ cm$$



$$\left.\begin{array}{l} \sphericalangle SFG \cong \sphericalangle SEG\ (pravi) \\ \sphericalangle CSF \cong \sphericalangle GSE\ (zajed.) \end{array}\right\} \xRightarrow{2.st.(UU)}$$

$$\Rightarrow \Delta SCF \cong \Delta SGE$$

$$\Rightarrow SF : SE = CF : GE$$

$$\Rightarrow 4 : 3 = 4 : GE$$

$$\Rightarrow GE = 3\ cm$$

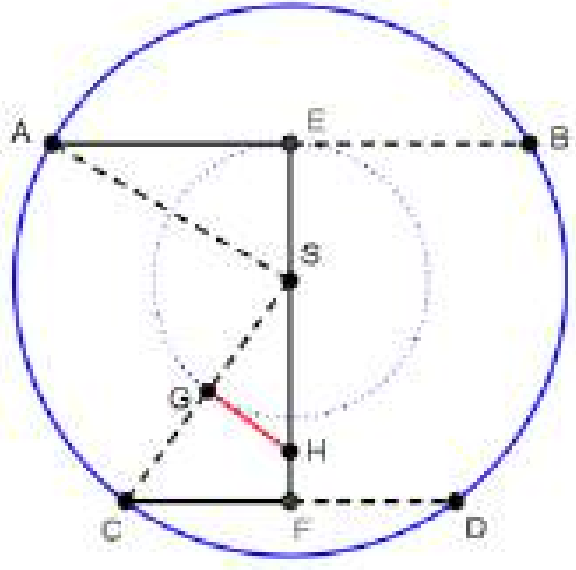
$$\Rightarrow AG = 1\ cm$$

AUTOR: Bogdan Đurić

REŠENJE

Na osnovu crteža i datih podataka, imamo da je:

$$\left.\begin{array}{l} AE = 8\ cm \wedge r^2 = AE^2 + SE^2 \Rightarrow r = 10\ cm \\ CF = 6\ cm \wedge SF^2 = r^2 - CF^2 \end{array}\right\} \Rightarrow SF = 8\ cm$$



$$\left.\begin{array}{l} \sphericalangle SGH \cong \sphericalangle SFC\ (pravi) \\ \sphericalangle HSG \cong \sphericalangle CSF\ (zajed.) \end{array}\right\} \xRightarrow{2.st.(UU)}$$

$$\Rightarrow \Delta SGH \cong \Delta SFC$$

$$\Rightarrow SF : SG = CF : GH$$

$$\Rightarrow 8 : 6 = 6 : GH$$

$$\Rightarrow GH = 4,5\ cm$$

$$\Rightarrow 2 \cdot GH = 9\ cm$$

AUTOR: Bogdan Đurić

REŠENJE

Na osnovu datih podataka, imamo da je:

$$\left.\begin{array}{l} s_1 = \dfrac{\mathcal{O}_1}{2} = 12\ cm \\ P_1 = r_1 \cdot s_1 \end{array}\right\} \Longrightarrow P_1 = 24\ cm^2 \Bigg\} \Longrightarrow$$

$$P_2 : P_1 = r_2^2 : r_1^2$$

$$\Longrightarrow P_2 : 24 = 9 : 4$$

$$\Longrightarrow P_2 = 54\ cm^2$$

AUTOR: Bogdan Đurić

NIVO

5 – ODLIČAN

AUTOR: Bogdan Đurić

REŠENJE

Na osnovu datih podataka, imamo da je:

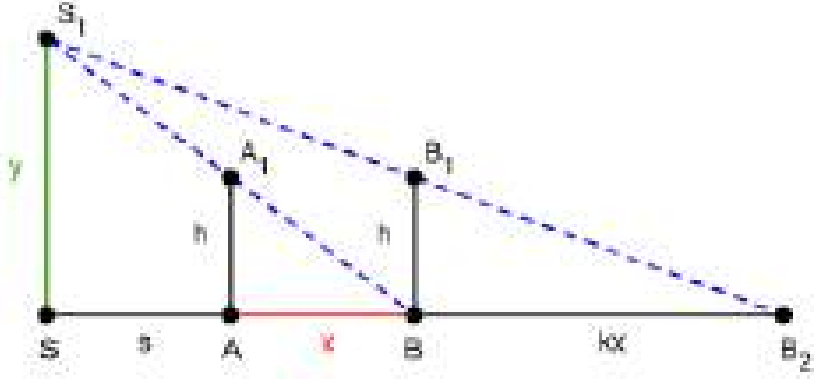
$$\left.\begin{array}{l} \Delta ABA_1 \sim \Delta SBS_1 \Rightarrow x:(s+x) = h:y \\ \Delta BB_2B_1 \sim \Delta SB_2S_1 \Rightarrow kx:(s+(k+1)x) = h:y \end{array}\right\} \Rightarrow$$

$$\Rightarrow x:(s+x) = kx:(s+(k+1)x)$$

$$\Rightarrow x = s\cdot(k-1)$$

$$\Rightarrow x = 7\cdot(2-1)$$

$$\Rightarrow x = 7\ m$$



AUTOR: Bogdan Đurić

REŠENJE

Na osnovu datih podataka, imamo da je:

$$\left.\begin{array}{l} s_1 = \dfrac{\mathcal{O}_1}{2} = 12\ cm \\ P_1 = r_1 \cdot s_1 \end{array}\right\} \Rightarrow r_1 = 2\ cm \Bigg\} \Rightarrow$$

$$P_2 : P_1 = r_2^2 : r_1^2$$

$$\Rightarrow 54 : 24 = r_2^2 : 4$$

$$\Rightarrow r_2 = 3\ cm$$

AUTOR: Bogdan Đurić

REŠENJE

Na osnovu datih podataka, imamo da je:

$$P_1 = \frac{d_1 \cdot d_2}{2}$$

$$P_1 = 6\ cm^2$$

$$P_1 = a_1 \cdot h_1$$

$$h_1 = 2,4\ cm$$

$$h_1 = 2r_1$$

$$r_1 = 1,2\ cm$$

$$a_1^2 = \left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2$$

$$a_1 = 2,5\ cm$$

$$P_2 : P_1 = r_2^2 : r_1^2$$

$$P_2 : 24 = 2,4^2 : 1,2^2$$

$$P_2 = 24\ cm^2$$

AUTOR: Bogdan Đurić

REŠENJE

Na osnovu datih podataka, imamo da je:

$$P_1 = \frac{d_1 \cdot d_2}{2}$$
$$P_1 = 6\ cm^2$$

$$a_1^2 = \left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2$$
$$a_1 = 2,5\ cm$$

$$P_1 = a_1 \cdot h_1$$
$$h_1 = 2,4\ cm$$

$$h_1 = 2r_1$$
$$r_1 = 1,2\ cm$$

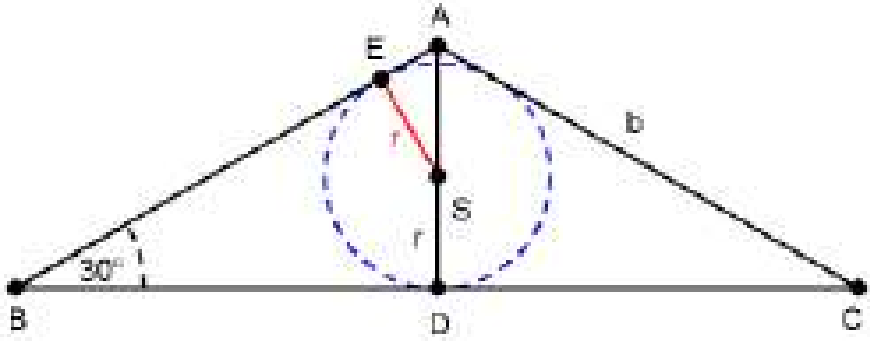
$$o_1 = 4a_1$$
$$o_1 = 10\ cm$$

$$o_2 : o_1 = r_2 : r_1$$
$$o_2 : 10 = 2,4 : 1,2$$
$$o_2 = 20\ cm$$

AUTOR: Bogdan Đurić

REŠENJE

Na osnovu datih podataka, imamo da je:



$$\Delta ABD\ karakter. \Longrightarrow$$

$$\Longrightarrow BD = \frac{b\sqrt{3}}{2} \wedge AD = \frac{b}{2}$$

$$\Delta SEA \sim \Delta BDA \Longrightarrow SE : BD = SA : BA \Longrightarrow r : \frac{b\sqrt{3}}{2} = \left(\frac{b}{2} - r\right) : b$$

$$\Longrightarrow r = b \cdot \frac{2\sqrt{3} - 3}{2} \Longrightarrow r = 3\ cm$$

AUTOR: Bogdan Đurić